Midterm I

Math 181B, UCSD, Spring 2018

Thursday, May 3rd, 3:30pm–4:50pm

Instructor: Eddie Aamari

- Write your PID, Name and Section legibly on your assignment.
- You may use a calculator (any type is fine), but no other electronic devices.
- You may not use your cell phone, tablet, or computer as a calculator.
- One handwritten page of notes allowed. (both sides OK, 8.5" by 11")
- Put away (and silence!) your cell phone and other devices that can be used for communication or can access the Internet.
- Show all of your work on your blue book; no credit will be given for unsupported answers.
- Justify your answers.

Exercise I

A Gallup Poll released in December 2010 asked 1019 adults living in the Continental U.S. about their belief in the origin of humans. These results, along with results from a more comprehensive poll from 2001 (that we will assume to be exactly accurate), are summarized in the table below

2010	2001	Response
38%	37%	Humans evolved, with God guiding
16%	12%	Humans evolved, but God had no part in the process
40%	45%	God created humans in the present form
6%	6%	Other/No opinion

At the level $\alpha = 5\%$, test whether or not beliefs on the origin of human life changed since from 2001 to 2010.

Turn Page

Exercise II

Recall that if $X \sim \mathcal{P}(\lambda)$ is Poisson distributed, $\mathbb{P}(X = x) = e^{-\lambda}\frac{\lambda^x}{x!}$ for all integer $x \geq 0$. Furthermore, $\mathbb{E}(X) = \lambda$ and $Var(X) = \lambda$.

1. Given $X_1, \ldots, X_n \sim_{iid} \mathcal{P}(\lambda)$, derive the asymptotic distribution of $\sqrt{n}(\bar{X}_n - \lambda)$.

2. If $X \sim \mathcal{P}(\lambda)$, give an explicit formula for $h(\lambda) = \mathbb{P}(X \geq 1)$.

3. For $0 < \alpha < 1$, build an asymptotic confidence interval of level $1 - \alpha$ for the parameter of interest $h(\lambda) = \mathbb{P}(X \geq 1)$.

Exercise III

We run a Gaussian test on a mean of interest μ in a population. The test is computed using n independent random variables $X_1, \ldots, X_n$ with Normal distribution $N(\mu, 1)$. We let $Z = \frac{\bar{X}_n - \mu_0}{1/\sqrt{n}}$. We want to test the one-sided hypotheses:

$$H_0\colon \mu = \mu_0, \quad H_1\colon \mu > \mu_0.$$

We let μ_1 denote the *true* mean if H_1 is true. We fix a confidence level α and we compute the critical value z^*_α such that $\mathbb{P}(Z > z^*_\alpha | H_0 \text{ is true}) = \alpha$. (for $\alpha = 5\%$, $z^*_\alpha = 1.64$)

1. What is the distribution of Z under H_0? And under H_1?

2. After defining what the power of a test is (in words), show that the power $1 - \beta$ of this test can be written as

$$1 - \beta = \mathbb{P}\left(Y > z^*_\alpha - \frac{\mu_1 - \mu_0}{1/\sqrt{n}}\right),$$

where Y has distribution $N(0, 1)$.

3. Write $\Delta = \mu_1 - \mu_0$ for the difference of means. Study qualitatively the variations (*i.e.* increasing/decreasing/constant) of the power of the test when:

   (a) α varies;

   (b) Δ varies;

   (c) n varies.

4. For $\alpha = 5\%$ and $n = 30$, find the smallest difference of means Δ that we can differentiate in the test, while guaranteeing a power $1 - \beta \geq 80\%$.